Robust Principal Component Analysis for Iterative Learning Control of Precision Motion Systems with Non-repetitive Disturbances

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Abstract—In precision motion systems, the same desired trajectory may have to be repeatedly followed. In such cases, iterative learning control (ILC) is a useful strategy to improve the tracking performance at every iteration cycle. The fundamental assumption is that the error is due to repetitive disturbances. In practice, however, non-repetitive disturbances may also be present, and non-repetitive and repetitive disturbances may possess common frequency components. If non-repetitive disturbance effects enter the learning loop, the performance of ILC may be degraded. This paper studies the problem of robust ILC in the presence of non-repetitive disturbances. An optimization based time-domain Q-filtering technique is presented to prevent non-repetitive disturbances from entering the ILC learning loop. More precisely, we apply the robust principal component analysis (RPCA) to filter out non-repetitive effects from the error signals. The effectiveness of the proposed method is demonstrated on a laboratory setup to emulate precision motion control stages of a wafer scanner. The method is also applicable to a broad class of precision motion systems.

I. INTRODUCTION

Iterative learning control (ILC) [1] has been extensively used in precision motion systems that repeatedly execute the same task. An essential premise of ILC is that the tracking errors of the controlled system are due to repetitive disturbances. Based on this idea, the errors from previous iterations can be incorporated to generate a feedforward signal to improve the system performance in the next iteration. In industrial practice, however, non-repetitive disturbances such as the force ripple of linear motors or other environmental vibrations can also affect the system behaviors. While the frequency characteristics of these non-repetitive signals are mostly fixed, their amplitudes and initial phases usually change from one trial to another. This will greatly limit or even degrade the achievable ILC performance.

With regard to this problem, a number of approaches have been proposed to improve the ILC robustness in the presence of non-repetitive disturbances. One popular approach is based on the high-order ILC [2], where they try to construct a disturbance observer in the iteration-domain. While this method can effectively handle non-repetitive disturbances, it may slow the convergence rate of ILC due to the high-order iteration-domain dynamics. Another approach is to design a Q-filter for filtering out the non-repetitive effects from the feedforward signal. Examples of this kind include the quadratic-criterion-based ILC [3], the selective ILC [4], and the ILC with a wavelet filter [5].

In this paper, the aforementioned problem is addressed by designing a new type of time-varying Q-filter based on the robust principal component analysis (RPCA) [6]. Broadly speaking, the idea behind the proposed method is to first extract the principal components of non-repetitive disturbances in the time-domain, and then recover the repetitive disturbances by subtracting those components from the feedforward signal. Comparing with other previous works, the proposed method stands out by the following two robustness features:

- As the filter is designed in the time-domain and not in the frequency-domain, it is expected to achieve superior performance when the frequency characteristics of non-repetitive disturbances are overlapped with that of repetitive disturbances.
- The proposed method is robust to variations of signal-to-noise ratio (SNR). This property is essential since the ratio of repetitive signal power to non-repetitive signal power is usually trajectory-dependent or iteration-dependent.

In the following section, the ILC formulation for systems with both repetitive and non-repetitive disturbances is presented. Then, the Q-filtering problem is casted into the RPCA framework and solved by a sequence of convex programs. The performance of the proposed approach is experimentally evaluated on a laboratory testbed wafer scanner. The results show that the proposed method significantly reduces the tracking error and improves the performance robustness compared to the standard ILC.

II. PRELIMINARIES AND PROBLEM FORMULATION

Fig. 1 shows a standard ILC scheme with reference update. The signals \( y_d(k), y_j(k), r_j(k), e_j(k), u_j(k), \) and \( d_j(k) \) denote the output reference, the system output, the tracking error, the control input, and the disturbance signal at the time step \( k \) in the \( j \)-th iteration, respectively. The corresponding ILC law of this scheme is:

\[
r_j(k) = Q(q)(r_{j-1}(k) + L(q)e_{j-1}(k + m))
\]

where \( q \) is the one-step advance operator and \( m \) is the delay steps in the controlled system. \( Q(q) \) and \( L(q) \) are, respectively, the robustness filter (a.k.a. the Q-filter) and the learning filter that will be designed later in this section. Then, by defining the feedforward signal prior to Q-filtering as \( r_f^p(j) \), the ILC law is further simplified to \( r_j(k) = Q(q)r_f^p(j) \).
The Q-filter is usually set as a lowpass filter for improving the stability robustness of the system.

Although the ILC can significantly reduce repetitive errors, it may also adversely affect the tracking performance by mistakenly learning non-repetitive errors. For example, consider a sinusoidal non-repetitive disturbance \(d_n(k) = \sin(\alpha_0)\) with a random initial phase \(\phi_j\) in each iteration and with a fixed frequency \(\omega_0\) within the Q-filter bandwidth. Since the frequency response of the Q-filter at \(\omega_0\) is approximately one, the error dynamics in (3) can be simplified to:

\[
e_{j+1}(k) = (1 - q^mLT)e_j(k) - (1 - T)P(d_{nj+1}(k) - d_n(k))
\]

where the difference between \(d_{nj+1}(k)\) and \(d_n(k)\) is:

\[
d_{nj+1}(k) - d_n(k) = 2\cos(\alpha_0 + \frac{\phi_{nj+1} - \phi_n}{2})
\]

Therefore, the standard ILC will amplify the errors induced by the non-repetitive disturbances whenever the phase difference between two sinusoids satisfies \(\left|2\cos(\alpha_0 + \frac{\phi_{nj+1} - \phi_n}{2})\right| > \frac{1}{2}\). With regard to this problem, we aim to attenuate these adverse effects of non-repetitive disturbances and to enhance the ILC robustness.

III. OPTIMIZATION BASED Q-FILTER DESIGN

The designed Q-filter is based on robust principal component analysis (RPCA), which is a variation of the commonly used tool principal component analysis (PCA). Recently, an increasing number of researchers in the field of computer vision become interested in applying RPCA to problems with grossly corrupted measurements. Some successful applications include face recognition [9] and video surveillance [10].

The common nature of these applications is that the measurements can be stacked as a data matrix:

\[
R = \begin{bmatrix} R_1 & R_2 & \cdots & R_M \end{bmatrix} \in \mathbb{R}^{N \times M}
\]

The objective of the standard PCA is to find a low-rank matrix \(H\) (said, with rank \(\ell\) that best represents the observations in the 2-norm sense:

\[
\min_{H,S} \| S \|_2 \text{ subject to } \|H + S - R\|_2 \leq \ell
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where $S$ is a noise term that describes the difference between $R$ and $H$. Due to the 2-norm objective, the standard PCA tends to achieve good performance when the noises are Gaussian. However, it may not work correctly in the case where the observations are grossly corrupted [11]. Therefore, [6] presented a convex program called principal component pursuit (PCP) to solve such robust PCA (RPCA) problem. PCP formulation is as follows:

$$\min_{H,S} \|H\|_* + \lambda \|S\|_1$$
subject to $H + S = R$ \hspace{1cm} (7)

where $\| \cdot \|_*$ is the nuclear norm and $\lambda$ is a design parameter. In [6], it is suggested to set the parameter as $\lambda \approx 1/\sqrt{\max(N,M)}$, while the exact value may need to be fine tuned for different applications. Note that this convex optimization problem can be solved efficiently using the algorithm presented in [12].

The Q-filtering problem is then formulated into RPCA framework. Recall the example in Sec. II where the system is affected by a sinusoidal non-repetitive disturbance with its frequency within the Q-filter bandwidth. Suppose there is a Q-filter that can perfectly filter out this non-repetitive signal without influencing the repetitive signal, the ILC error dynamics in (3) becomes:

$$e_{j+1}(k) = Q(1 - q^nLT)e_j(k) + (1 - Q)(1 - T)(y_d(k) - Pd_i(k)) - P(1 - T)d_{b,i+1}(k)$$

It is seen that the signal induced by the non-repetitive disturbance (i.e., the last term) would remain a sinusoid in every iteration, whereas the signal induced by repetitive effects (i.e., the first two terms) could have different “patterns” from one iteration to another. Therefore, it is natural to think of non-repetitive errors in all iterations as a matrix with low-dimensional principal components, and to think of repetitive errors as a noise term that “contaminates” the data matrix. This allows to use RPCA to decouple non-repetitive effects from repetitive effects.

Note that although the “patterns” of non-repetitive disturbances remain the same in the iteration-domain, they do not necessarily correspond to a single principal direction in the vector space $\mathbb{R}^N$ ($N$ is the length of trajectory). This is because the initial phases of the non-repetitive disturbances usually change from one trial to another. For this reason, it becomes necessary to define a time shifting operator to align the signals. Fig. 2 illustrates the time shifting operator $f : \mathbb{R}^K \rightarrow \mathbb{R}^N$ in detail, where $K$ is an integer that is assumed to be larger than $N$. With this operator, it becomes possible to design a Q-filter using a modification of RPCA. The algorithm proceeds as follows:

1) Measure $M - 1$ trials of error signals with length $K$ without applying any control action. Each signal is represented as a vector $R_i \in \mathbb{R}^K$, where $i \in \{1,2,\ldots,M - 1\}$.

2) In each trial of ILC, construct (or update) the vector $R_M = [r_p^o(1), r_p^o(2), \ldots, r_p^o(N)]^T \in \mathbb{R}^N$.

3) Solve the modified RPCA problem for Q-filtering:

$$\min_{H,S,t,b} \|H\|_* + \lambda \|S\|_1$$
subject to $H_i + S_i = f(R_i, \tau_i), \forall i \in \{1,\ldots,M - 1\}$

$$H_M + S_M = R_M - b$$

where $b$ is a scalar variable accounts for the bias (see Fig. 3 for the details).

4) Design the Q-filter as:

$$r_p^i(k) = Q\mathcal{P}\{r_p^i(k)\}$$

where $\mathcal{P}\{\cdot\}$ is a transformation defined by $\mathcal{P}\{r_p^i(k)\} = S_d^i(k) + b^*. S_M^i$ and $b^*$ are the optimizers of the RPCA problem described in (8). $Q$ is an additional lowpass filter for improving the system robustness.

5) Run an iteration of ILC, and continue with Step 2.

Note that the new decision variable $\tau$ introduced into RPCA makes the optimization problem non-convex. Thus the sequential convex programming (SCP) [13] is applied to approach this problem. More precisely, we first initialize the time shift $\tau = \tau^0$ by using cross-correlation function for each pair of signals. Then linearize the constraints in (8) around $\tau^0$ to obtain the approximate convex program:

$$\min_{H,S,t,b} \|H\|_* + \lambda \|S\|_1$$
subject to $H_i + S_i = J_i(\tau_i - \bar{\tau}) + f(R_i, \bar{\tau}), \forall i$

$$H_M + S_M = R_M - b$$

$$\|\tau - \bar{\tau}\|_2 \leq \varepsilon$$

where $J_i = \nabla_{\tau_i}f(R_i, \bar{\tau})$ is the Jacobian matrix of the time-shifting operator. As the constraint only holds locally, a variable $\varepsilon$ is introduced to restrict the size of the decision variable space. Once the optimal solution of (8) is found, the optimizer $\tau^*$ can be used to update the linearized point by $\tau = \tau^*$. By iteratively solving the problem (9), the
solution of (8) can be found. Convergence properties of such approximation techniques was discussed in a related work of [9].

IV. CASE STUDY

This section describes the application of the proposed Q-filtering technique to the wafer scanner control problem. The method is also applicable to a broad class of precision motion systems.

A. Experimental Setup

A wafer scanner is a machine that performs the essential photolithography steps in the manufacture of integrated circuits. It consists of a light source, a reticle stage, several projection lenses and a wafer stage. Among them the wafer stage and the reticle stage are both high precision motion systems that carry a silicon wafer and a mask with designed circuits patterns. A laboratory testbed wafer scanner is shown in Fig. 4. In this system, both stages are driven by three-phase linear motors. The stage position is measured by a laser interferometry system with a sampling rate of 2.5kHz. The system has a non-repetitive disturbance at 18.32Hz caused by the force ripple. As the non-repetitive disturbance in the system has relatively small power, an artificial disturbance at the same frequency is added for amplifying the effects. All the experiments in this paper are performed on the reticle stage. The measured and the identified closed-loop frequency response from the output reference \( y_d(k) \) to the system output \( y(k) \) are shown in Fig. 5. A baseline controller used in the system is a PID controller. The L-filter is designed to be the inverse of the identified closed-loop transfer function. The baseline Q-filter is set as a lowpass filter with the cutoff frequency at 150Hz.

B. Experimental Results

The proposed method is tested for two different trajectories in Fig. 6. The first trajectory is a second order smooth time-optimal [14] point-to-point scanning trajectory with a maximum velocity of 0.1875m/sec and a maximum acceleration of 5m/sec². The second trajectory consists of two 2Hz sinusoids with a maximum velocity of 0.225m/sec. The shaded areas in Fig. 6 indicate intervals where the ratios of repetitive signal power to non-repetitive signal power are likely to be large. For example, consider the point-to-point scanning trajectory. It is natural to expect that the system will have large repetitive errors whenever the acceleration reference has large magnitude. Whereas in the constant speed phase or the stationary phase, the errors caused by trajectory tracking is likely to close to zero and the non-repetitive effects will dominate the errors. Such prior knowledge about the trajectory is important since it allows to estimate how sparse the component \( S_M \) will be. The proposed algorithm is expected to provide more accurate decomposition in the
case when repetitive errors only dominate a small fraction of the trajectory rather than the whole trajectory.

Fig. 7 shows the results of the point-to-point scanning trajectory tracking task. The time-domain feedforward signals before and after applying RPCA are shown. It is seen that the behaviors of these signals match are as expected, namely repetitive errors only dominate a small fraction of the trajectory. Also, it is seen that RPCA can effectively filter out the non-repetitive errors caused by the force ripple regardless of SNR. This property is essential since the ratio of repetitive signal power to non-repetitive signal power is usually trajectory-dependent or iteration-dependent.

Fig. 8 shows the position errors over 15 iterations of ILC with and without the proposed Q-filter. It is seen that both methods eliminate most repetitive errors in one iteration. In the 2-nd iteration to the 15-th iteration, the proposed method outperforms the standard ILC in accuracy and robustness. More precisely, the mean of errors is reduced by 19.2%, and the standard deviation is reduced by 69.2%.

The performance of the proposed method and the standard ILC in frequency-domain are compared in Fig. 9 in terms of spectra around 18.32Hz, and in Table I quantitatively. Note that the error variance of the standard ILC is much larger than that of the proposed method. In fact, even the maximum value of the proposed algorithm is smaller than the mean value of the standard ILC.

To further test the capability of the proposed method on filtering non-repetitive effects, a more challenging task of following a sinusoidal reference trajectory is considered. The main difficulty is that the repetitive errors and the non-repetitive errors may have the same order of magnitude over the whole trajectory. Fig. 10 shows the feedforward signal before and after applying RPCA. Note that RPCA is able to filter out the non-repetitive part and preserve the repetitive one. Fig. 11 shows the iteration-domain error convergence profiles for both the proposed method and the standard ILC. Although the performance improvement is not as significant as the previous task, the proposed algorithm still give an 11.2% improvement in the mean value, and a 50.8% improvement in standard deviation. The error spectrum in Fig. 12 shows that the proposed ILC scheme is able to enhance the performance robustness of ILC in the presence of non-repetitive disturbances. A quantitative comparison is shown in Table I.
This paper presented a time-domain Q-filtering technique in ILC for robustness enhancement in the presence of non-repetitive disturbances. The Q-filtering problem in the RPCA framework has been formulated so that it can be solved efficiently by a sequence of convex optimization problems. Experimental results show that the proposed method can successfully prevent the non-repetitive disturbances from entering the ILC learning loop, and thus can significantly improve the performance robustness of ILC.

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REFERENCES